Name: _____

- This packet is due at **12:00 pm PDT (noon) on Friday, April 30**.
- Neatly fill out this packet either directly or on your own paper. Then upload your answers to Canvas. Please clearly number each problem and keep them in order.
- For each problem in this packet, **organize and show all of your work and justification** for your answer.
- Make sure that it is clear what your final answer is. This may mean boxing/circling it or it may mean doing neither.
- If you are required to draw a graph of a function, each axis must be labeled and given a scale.
- I will not be grading every individual problem, but answers will be available on Canvas after the due date.
- I am happy to help you however I can. Here are some options:
  - Come to my office hours (see syllabus).
  - Email me directly at gknapp4@uoregon.edu

1. Use the limit comparison test to determine whether each of the following series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3-2}}$

$\frac{1}{n^{3/2}}$

$\lim_{n\to\infty} \frac{n^{3/2}}{(n^3-2)^{1/2}} = \lim_{n\to\infty} \left(\frac{n^3}{n^3-2}\right)^{1/2}$

$= \left(\lim_{n\to\infty} \frac{n^3}{n^3-2}\right)^{1/2}$

converge

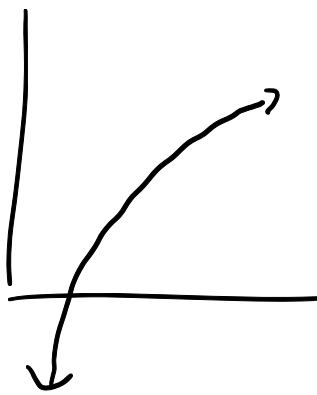
$\log(n) << n^{\varepsilon}$

converge

(c) $\sum_{n=1}^{\infty} \left(\frac{\ln(n)}{n}\right)^2$

$a_n = \frac{\ln(n)^2}{n^2}$

$b_n = \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}$



$\sum \frac{1}{n}$ diverges

(d) $\sum_{n=1}^{\infty} \frac{1}{n^2 - n\sin(1/n)}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n - 3^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n - 1}{5^n}$

2. State whether the following series converges absolutely, conditionally, or not at all.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+3}$

(1) Div. test
(2) Abs. conv.
(3a) If abs. conv. ✓ done
(3b) If abs. conv. X → A.S.T.

(b) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}$

A.C.T.

$$\sum_{n=1}^{\infty}\left|\frac{(-1)^{n+1}}{n!}\right| = \sum_{n=1}^{\infty}\frac{1}{n!}$$

converges

$$\frac{1}{n\cdot(n-1)\cdots 2\cdot 1} = \frac{1}{n!} \le \frac{1}{2^{n-1}}$$

$n\cdot(n-1)$: $n\ge 2$, $(n-1)\ge 2$; $\cdots 2\cdot 1$: $2\ge 2$, $1\ge 2$

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{1+n^4}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$

(e) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{n+3}}{n}$

uses 2^x is cts.

(f) $\sum_{n=1}^{\infty}(-1)^{n+1}2^{1/n}$

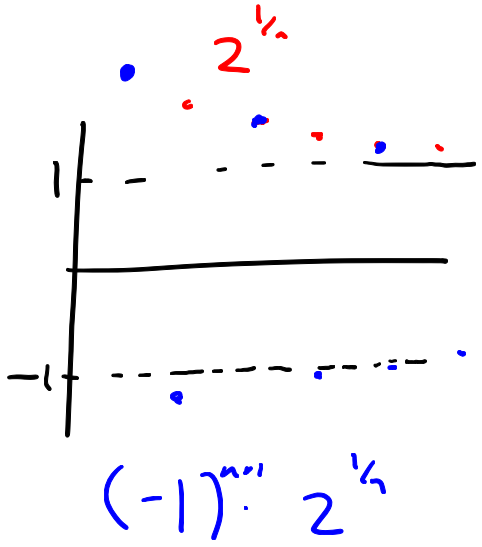
$\lim_{n\to\infty} \frac{1}{n}$

$\lim_{n\to\infty} 2^{1/n} = 2^{\lim_{n\to\infty}\frac{1}{n}} = 1$

$\lim_{n\to\infty} (-1)^{n+1} 2^{1/n}$ DNE

$\Rightarrow$ By div. test,

$\sum_{n=1}^{\infty} (-1)^{n+1} 2^{1/n}$ diverges



3. In each of the following problems, use the estimate $|R_k| \leqslant b_{k+1}$ to find a value of k that guarantees that the sum of the first k terms of the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}b_n$ differs from the infinite sum by at most the given error. Calculate the partial sum S_k for this k.

(a) $b_n = 1/\sqrt{n}$, error $< 10^{-3}$

ineq. remains ↔ applying an inc. fn.

flips ↔ applying a dec.

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$$

$|R_k| \leq |b_{k+1}|$

Goal: $|R_k| < 10^{-3}$

If $|b_{k+1}| < 10^{-3}$, we're good ✓

$$\frac{1}{\sqrt{k+1}} < 10^{-3} \longleftrightarrow \frac{1}{10^{-3}} < \sqrt{k+1}$$

$$10^3$$

(b) $b_n = 1/2^n$, error $< 10^{-6}$

x^2 inc.

$(10^3)^2 < k+1$

$10^6 - 1 < k$

1,000,000 $= 10^6 = k$

"sum from 1 to 1,000,000 of $(-1)^{n+1}/\sqrt{n}$"

$$\sum_{n=1}^{1000000}\frac{(-1)^{n+1}}{\sqrt{n}} \approx$$

(c) $b_n = \ln\left(1 + \frac{1}{n}\right)$, error $< 10^{-3}$

Goal: $|R_k| < 10^{-3}$

Know: $|R_k| \leq |b_{k+1}|$

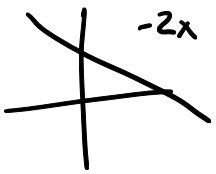
If $|b_{k+1}| < 10^{-3}$, we're good

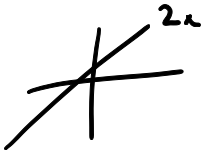
$$\ln\left(1 + \frac{1}{k+1}\right) < 10^{-3}$$

$$1 + \frac{1}{k+1} < e^{0.001}$$

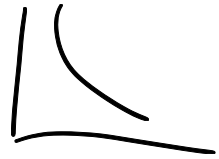
$$\frac{1}{k+1} < e^{0.001} - 1 \iff k+1 > \frac{1}{e^{0.001}-1}$$

$$\frac{1}{e^{0.001}-1} < k+1$$





$\frac{1}{x}$ is dec.



(d) $b_n = 1/n^2$, error $< 10^{-6}$

$$10^6 < 2^{k+1}$$

$\log_b(x)$ is inc. when $b > 1$

$$\log_2(10^6) < k+1$$

4. Suppose that a_n is a sequence of positive real numbers and that $\sum_{n=1}^{\infty} a_n$ converges. Suppose that b_n is an arbitrary sequence of ones and minus ones. Does $\sum_{n=1}^{\infty} a_n b_n$ necessarily converge?

~~A.S.T.~~

A.C.T. ← absolute convergence test

↳ If $\sum_{n=n_0}^{\infty} |x_n|$ converges, then $\sum_{n=n_0}^{\infty} x_n$ also converges.

Want to know about $\sum a_n b_n$ $\sum a_n$

Q. What do we know about $\sum |a_n b_n|$?